2 The Likelihood

The log–likelihood and its first and second derivatives are given by

$$L(\boldsymbol{\beta}) \equiv \log p(\boldsymbol{y}, \boldsymbol{x} \mid \boldsymbol{\beta}) = -\sum_{i=1}^{n} \log\left(1 + \exp\{-\eta_i\}\right)$$

$$L'(\boldsymbol{\beta}) \equiv \frac{\partial}{\partial \beta_j} L(\boldsymbol{\beta}) = \sum_{i=1}^{n} \frac{x_{ij} y_i}{1 + \exp \eta_i}$$

$$L''(\boldsymbol{\beta}) \equiv \frac{\partial^2}{\partial \beta_j^2} L(\boldsymbol{\beta}) = -\sum_{i=1}^{n} x_{ij}^2 \frac{\exp \eta_i}{(1 + \exp \eta_i)^2}$$

where $\eta_i = y_i \left(\beta_0 + \sum_{j=1}^{k} \beta_{ij} x_{ij}\right)$ and $y \in \{-1, 1\}$ denotes case/control status.