Color Edge Detection Using Multiscale Quaternion Convolution

Jiangyan Xu,[1] Linning Ye,[2] Wang Luo[1]

[1] School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu 610054, China

[2] School of Economic Information Engineering, Southwestern University of Finance and Economics, Chengdu 610074, China

Received 21 September 2008; revised 22 July 2010; accepted 28 September 2010

ABSTRACT: This article presents a novel color edge detection algorithm based on quaternion convolution. A set of multiscale quaternion masks are presented, and the filtered results are multiplied at adjacent scales using dot product to enhance the edge structures while diluting noise. Experimental results show that the proposed method is efficient and robust for color images. Int J Imaging Syst Technol, 20, 354–358, 2010; Published online in Wiley Online Library (wileyonlinelibrary.com). DOI 10.1002/ima.20258

Key words: quaternion convolution; edge detection; dot product; color image; multiscale

I. INTRODUCTION

Edge detection is a common image processing task that often forms the initial stage of automated image interpretation. In many situations, strong edges between colors become very weak in luminance and are difficult to detect. The advantage of color edge detection schemes over gray-level approaches is easily demonstrated by considering the fact that those edges existing at boundary between regions of different colors cannot be detected in gray-level images if there is no change in intensity. The techniques used for color edge detection can be subdivided into two classes: the monochromatic-based techniques and vector-valued techniques (Koschan and Abidi, 2005). Up to now, color edge detection has been an active research area for more than 3 decades.

One type of the color edge detection methods is the reduced ordering (R-Ordering) used by the vector order statistics edge detector (Trahanias and Venetsanopoulos, 1993). Several other color edge detectors based on the vector order statistics were proposed, of which the minimum vector dispersion (MVD) was shown to be the most effective. However, MVD is unable to provide an estimate of edge direction. The entropy index can also be used in edge detection. The edge detection algorithm based on local, nonparametric estimation of image density has been presented by Economou (2004), in which the edges are regarded as a set of points separating two coherent regions. This method locates and estimates the value of the density minima at region boundaries as a measure of edge strength.

Color image filtering based on separately processing color components (red, green, and blue in the case of RGB images) limits the type of filter that can be realized. It was reported that color images may be transformed using a quaternion Fourier transform (Sangwine, 1998; Pei and Cheng, 1999; Sangwine and Ell, 2000; Jin and Li, 2007; Said et al., 2008). To study the implementation of filters using quaternion convolution for edge detection, quaternion convolution will be briefly introduced in the next section.

This article presents a new color edge detector based on quaternion convolution; at the same time the edge direction is computed for modulus maximum suppression. To enhance edge structures while diluting noise, the filtered results at adjacent scales are multiplied using dot product. This article is organized as follows: the properties of quaternion convolution are presented in Section II; Section III presents the proposed edge detection method, and the experimental results are shown in Section IV; Section V draws the conclusion.

II. PROPERTIES OF QUATERNION CONVOLUTION

To effectively compute the multidimensional color data, an efficient expression of the color data is necessary. The algebra of the quaternion is the generalization of complex numbers (Pei and Cheng, 1999).

Considering a four-dimensional real-valued data set $Q = \{(q_0(n), q_1(n), q_2(n), q_3(n))\}_{n=1}^{N}$, a quadruple data point $(q_0(n), q_1(n), q_2(n), q_3(n))$ can be expressed as a quaternion number $\hat{q}(n)$

$$\hat{q}(n) = q_0(n) + q_1(n) \cdot i + q_2(n) \cdot j + q_3(n) \cdot k, \qquad (1)$$

where i, j, and k denote the operation units of quaternion number with rules of operation as follows:

Correspondence to: Jiangyan Xu; e-mail: xujiangyan@126.com

$$
\begin{aligned}
&i^2 = j^2 = k^2 = -1 \\
&i \cdot j = k \quad j \cdot k = i \quad k \cdot i = j \\
&j \cdot i = -k \quad k \cdot j = -i \quad i \cdot k = -j\,.
\end{aligned} \tag{2}
$$

Any vector $v \in R^3$ can be expressed as a quaternion with q_0 set to be zero. For example, a color value (R,G,B) can be shown as a quaternion with $q_1 = R$, $q_2 = G$, $q_3 = B$, and $q_0 = 0$. Also, a quaternion can be represented as $\hat{q}\,(n) = \langle \alpha\,,\beta \rangle$, where $\alpha = (q_1\,(n), q_2\,(n), q_3\,(n)) = V\,(\hat{q})$ and $\beta = q_0\,(n) = S\,(\hat{q})$. Some important properties of quaternion operations are given as follows:

1. Based on the cross product ($\times$) of vector space, we define multiplication of two quaternions, $\hat{q}$ and $\hat{q}'$, as

$$
\begin{aligned}
\hat{q} \cdot \hat{q}' &= \langle \alpha, \beta \rangle \cdot \langle \alpha', \beta' \rangle \\
&= \langle \alpha \times \alpha' + \beta \cdot \alpha' + \beta' \cdot \alpha, \beta \cdot \beta' - \alpha \cdot \alpha' \rangle
\end{aligned} \tag{3}
$$

2. The conjugate $\hat{q}^*$ of $\hat{q}$ is defined as

$$
\hat{q}^* = -\langle \alpha, \beta \rangle = q_0 - (q_1 \cdot i + q_2 \cdot j + q_3 \cdot k) \tag{4}
$$

3. The norm of the quaternion is denoted as $\|\hat{q}\|^2 = \hat{q} \cdot \hat{q}^*$.

III. THE PROPOSED METHOD FOR COLOR EDGE DETECTION

It is well known that we can implement image filtering by convolving a filter "mask" with an image. Therefore, it is natural to consider convolution with a quaternion-valued mask. From Eq. (2), we know that quaternion multiplication is not commutative. As a result, the general case of a quaternion-based filter requires both left mask and right mask. The value within the mask is inspired by the Prewitt detector (Pratt, 1991) and following masks define new filters for detecting horizontal and vertical edges, respectively.

$$
\begin{aligned}
q_h f(x,y) &= [R \quad 0 \quad R^*][\cdots \quad f(x-1,y) \quad 0 \quad f(x+1,y) \quad \cdots] \\
&\quad\; [R^* \quad 0 \quad R] \\
q_v f(x,y) &= \begin{bmatrix} R \\ 0 \\ R^* \end{bmatrix} \begin{bmatrix} \vdots \\ f(x,y+1) \\ 0 \\ f(x,y-1) \\ \vdots \end{bmatrix} \begin{bmatrix} R^* \\ 0 \\ R \end{bmatrix}
\end{aligned} \tag{5}
$$

where $f\,(x,y)$ is the original color image pixel. The value of R is given by $R = e^{\mu\theta}$, and μ is a three-dimensional unit vector that is represented by a unit pure quaternion. In this article, let $\mu = (i+j+k)/\sqrt{3}$ and $\theta = \pi/2$. The new filter for color edge detection uses a rotation in color space about an axis as $r = g = b$, the so-called "gray line" of RGB space on which lie all achromatic pixel values (Jin and Li, 2007). Any rotation about this axis maps a color pixel value into another color pixel value with the same luminance but different hue.

The quaternion operator RXR^* defines a rotation of X in the RGB space about the axis μ. The color generated by the filter at an edge between two colors is mid-way between the colors in the hue sense. Reversing the sense of the filter by interchanging R and R^* in the masks changes the directions in which two colors are rotated (Sangwine, 1998).

Suppose the filtered coefficient results in Eq. (5) are $q_h f(x,y) = [q_{hi}\, f(x,y)\ q_{hj}\, f(x,y)\ q_{hk}\, f(x,y)]^T$ in the horizontal direction, and the coefficients are $q_v\, f(x,y) = [q_{vi}\, f(x,y)\ \ q_{vj}\, f(x,y)\ \ q_{vk}\, f(x,y)]^T$ in the vertical direction. The modulus in the two directions are defined as

$$
\begin{aligned}
q_1 f(x,y) &= \sqrt{q_{hi}^2 f(x,y) + q_{hj}^2 f(x,y) + q_{hk}^2 f(x,y)} \\
q_2 f(x,y) &= \sqrt{q_{vi}^2 f(x,y) + q_{vj}^2 f(x,y) + q_{vk}^2 f(x,y)}
\end{aligned}\,. \tag{6}
$$

To thin the edge magnitude, we must assign signs to the modulus in the horizontal and vertical directions, respectively. There are many methods to achieve this purpose. After filtering an image using Eq. (5), the real part remains to be zero. To express the relative intensity between, for example, $f\,(x-1,\,y)$ and $f\,(x+1,\,y)$ in the horizontal direction, the signs for edge direction can be obtained as

$$
\begin{aligned}
s_1(x,y) &= \operatorname{sgn}(S([R \quad 0 \quad R^*][f(x-1,y) \quad 0 \quad f(x+1,y)])) \\
s_2(x,y) &= \operatorname{sgn}\left(S\left(\begin{bmatrix} R \\ 0 \\ R^* \end{bmatrix}\begin{bmatrix} f(x,y+1) \\ 0 \\ f(x,y-1) \end{bmatrix}\right)\right).
\end{aligned} \tag{7}
$$

The angle with the horizontal direction is given by

$$
Af(x,y) = \arg\tan\left(\frac{s_2(x,y) q_2 f(x,y)}{s_1(x,y) q_1 f(x,y)}\right). \tag{8}
$$

Then, the modulus is proportional to

$$
\begin{aligned}
&Mf(x,y) = \sqrt{q_1^2 f(x,y) + q_2^2 f(x,y)} \\
&= \sqrt{q_{hi}^2 f(x,y) + q_{hj}^2 f(x,y) + q_{hk}^2 f(x,y) + q_{vi}^2 f(x,y) + q_{vj}^2 f(x,y) + q_{vk}^2 f(x,y)}\,.
\end{aligned} \tag{9}
$$

The edge points are the points $(x,\,y)$, where the modulus $Mf\,(x,\,y)$ has local maxima in the direction given by $Af\,(x,\,y)$.

To suppress the noise and obtain multiscale edges, we introduce the multiscale quaternion edge detector. First of all, if we want to detect the vertical edges, for example, at scale m, we define a set of quaternion filters as follows:

$$
\begin{aligned}
\operatorname{mask}_l^1(R,R^*,m) &= \Big[R \quad \underbrace{0 \quad \cdots \quad 0}_{2m-1} \quad R^*\Big] \\
\operatorname{mask}_r^1(R^*,R,m) &= \Big[R^* \quad \underbrace{0 \quad \cdots \quad 0}_{2m-1} \quad R\Big]
\end{aligned}, \tag{10}
$$

where $m = 1{,}2{,}3{,}\cdots$, l denotes the left mask and r denotes the right mask.

Using these masks to convolute a row of pixels in the horizontal direction, we obtain the results as

$$
\begin{aligned}
&C_m^1 f(x) = \\
&\operatorname{mask}_l^1(R,R^*,m)\Big[f(x-m) \quad \underbrace{0 \quad \cdots \quad 0}_{2m-1} \quad f(x+m)\Big]\operatorname{mask}_r^1(R,R^*,m).
\end{aligned} \tag{11}
$$

From Eq. (11), we obtain the following equation:

$$\Bigg[\underbrace{R\ \cdots\ R}_{m}\ 0\ \underbrace{R^*\ \cdots\ R^*}_{m}\Bigg] f(x) \Bigg[\underbrace{R^*\ \cdots\ R^*}_{m}\ 0\ \underbrace{R\ \cdots\ R}_{m}\Bigg] = C_m^1 f(x) + C_{m-1}^1 f(x) + \cdots + C_1^1 f(x). \quad (12)$$

Therefore, we define the multiscale quaternion masks in the horizontal direction as

$$\begin{aligned} mq_l^1(R, R^*, m) &= \Bigg[\underbrace{R\ \cdots\ R}_{m}\ 0\ \underbrace{R^*\ \cdots\ R^*}_{m}\Bigg] \\ mq_r^1(R, R^*, m) &= \Bigg[\underbrace{R^*\ \cdots\ R^*}_{m}\ 0\ \underbrace{R\ \cdots\ R}_{m}\Bigg] \end{aligned}. \quad (13)$$

Similar to Eq. (13), the multiscale quaternion masks in the vertical direction are defined as

$$\begin{aligned} mq_u^1(R, R^*, m) &= \Bigg[\underbrace{R\ \cdots\ R}_{m}\ 0\ \underbrace{R^*\ \cdots\ R^*}_{m}\Bigg]^T \\ mq_d^1(R, R^*, m) &= \Bigg[\underbrace{R^*\ \cdots\ R^*}_{m}\ 0\ \underbrace{R\ \cdots\ R}_{m}\Bigg]^T \end{aligned}, \quad (14)$$

where u denotes the upper mask and d denotes the lower mask.

Using the masks shown in Eqs. (13) and (14) to convolute the image in the horizontal and vertical directions, we can compute the edge magnitude at different scales. By modulus maximum suppression, we obtain the thinned edges.

In the references (Bao and Zhang, 2003; Bao et al., 2005), a wavelet-based multiscale products thresholding scheme for noise suppression has been presented. By multiplying adjacent wavelet subbands, edges can be effectively distinguished from noise. However, in color image processing, vector-valued image functions are treated instead of scalar image functions. Therefore, we introduce the dot product here.

The dot product is fundamentally a projection. Suppose two vectors $\vec{a} = a_1 i + a_2 j + a_3 k$ and $\vec{b} = b_1 i + b_2 j + b_3 k$ are in R^3. Geometrically, the dot product is useful for finding the direction between arbitrary vectors in space. The dot product of the two vectors is the scalar given by

$$\vec{a} \bullet \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \|\vec{a}\|\|\vec{b}\| \cos\theta, \quad (15)$$

where the magnitudes are calculated using $\|\vec{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

When two vectors are around in the same direction, i.e., the angle is small, the dot product is large. After the image is filtered at different scales, we use the dot product to denote the correlation between the two filtered results.

When we use multiscale quaternion masks to convolute the image, Eq. (5) is rewritten as

$$\begin{aligned} q_m^1 f(x, y) &= mq_l^1(R, R^*, m) \\ &[\cdots\ f(x-1, y)\ \ 0\ \ f(x+1, y)\ \ \cdots] mq_r^1(R, R^*, m) \\ q_m^2 f(x, y) &= mq_u^2(R, R^*, m) \begin{bmatrix} \vdots \\ f(x, y+1) \\ 0 \\ f(x, y-1) \\ \vdots \end{bmatrix} mq_d^2(R, R^*, m). \end{aligned} \quad (16)$$

When the image is noisy, the correlation between the two results at adjacent scales will decrease. To exploit the interscale dependencies, we multiply the results using dot product at adjacent scales to enhance edge structures while weakening noise. For 2D images, the multiscale products have two components

$$\begin{aligned} P_m^1 f(x, y) &= q_m^1 f(x, y) \bullet q_{m+1}^1 f(x, y) \\ P_m^2 f(x, y) &= q_m^2 f(x, y) \bullet q_{m+1}^2 f(x, y) \end{aligned}, \quad (17)$$

where $\bullet$ denotes the dot product.

The support of an isolated edge increases by a factor of two across scale, and the neighboring edges interfere with each other at coarse scales (Bao and Zhang, 2003). In practice, it is sufficient to implement the multiplication at two adjacent scales. With the decreasing of the dependency between the two scales, the scalar $P_m^1 f(x,f)$ or $P_m^2 f(x,f)$ is assigned a minus sign if the direction is opposite. In this case, we can imagine the pixels are not edge points or the edge pixels are in thick noise. To suppress the noise, following equations are introduced:

$$\begin{aligned} Q_m^1 f(x, y) &= P_m^1 f(x, y) U\big(P_m^1 f(x, y)\big) \\ Q_m^2 f(x, y) &= P_m^2 f(x, y) U\big(P_m^2 f(x, y)\big) \end{aligned}, \quad (18)$$

where $U(t) = \begin{cases} 1 & t > 0 \\ 0 & t \le 0 \end{cases}$.

The signs for modulus maximum suppression are given by

$$\begin{aligned} s_1(x, y) &= \operatorname{sgn}\left(S\left(\sum_{w=m}^{m+1} mq_l^1(R, R^*, w)[\cdots f(x-1, y)\ 0\ f(x+1, y)\ \cdots]\right)\right) \\ s_2(x, y) &= \operatorname{sgn}\left(S\left(\sum_{w=m}^{m+1} mq_u^2(R, R^*, w) \begin{bmatrix} \vdots \\ f(x, y+1) \\ 0 \\ f(x, y-1) \\ \vdots \end{bmatrix}\right)\right). \end{aligned} \quad (19)$$

Then, the angle with the horizontal direction and the modulus are presented as

$$\begin{aligned} A_m f(x, y) &= \arg\tan\left(\frac{s_2(x, y) Q_m^2 f(x, y)}{s_1(x, y) Q_m^1 f(x, y)}\right) \\ M_m f(x, y) &= \sqrt{\left|Q_m^1 f(x, y)\right|^2 + \left|Q_m^2 f(x, y)\right|^2} \end{aligned}. \quad (20)$$

As in the Canny edge detector (Canny, 1986), an edge point is asserted wherever $M_m f(x, y)$ has a local maximum in the direction given by $A_m f(x, y)$.

IV. EXPERIMENTAL RESULTS

The performance of the proposed edge detector ($m = 2$ is chosen in the implementation) is compared against the MVD, the gradient, and the entropy detectors.

The results of these four operators on the noisy color ''peppers'' image are shown in Figure 1. Figure 1a shows the noisy ''peppers'' image ($\sigma = 35$), and Figure 1e presents the output of the proposed edge detector. Figures 1b–1d show the outputs of the vector gradient, MVD, and entropy edge detectors, respectively.

Comparing to other methods, the proposed method detects more detailed edge information and at the same time suppresses more noise, especially for the parts of the handles of the green peppers in

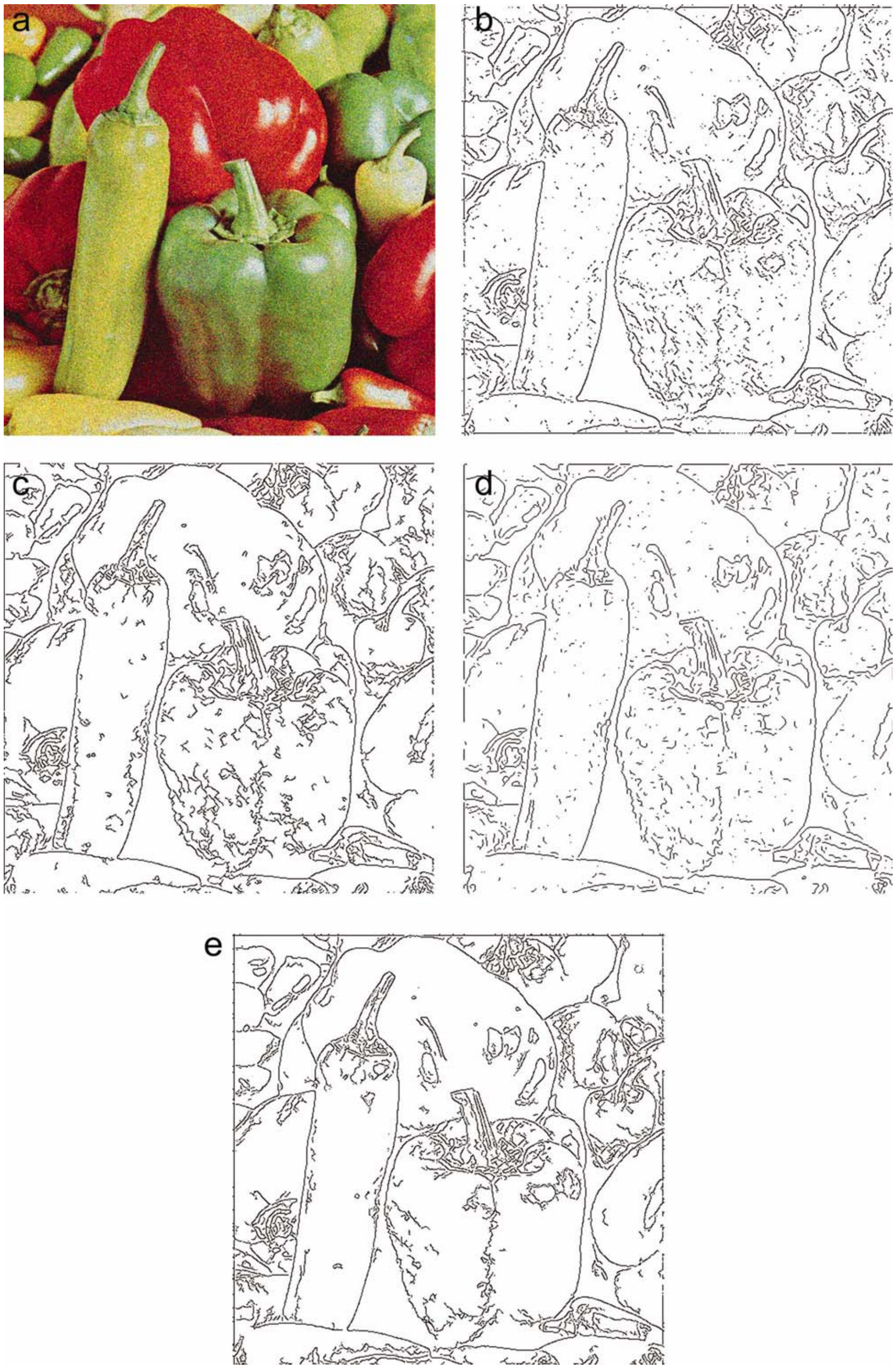

Figure 1. The original image and the edge maps of "peppers" (a). The original noisy color "peppers" image (b). Gradient (c). MVD (d). Entropy (e). Proposed method. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

the central of the image (Fig. 1e). In the top left corner, or the bottom of the image, the proposed algorithm also obtains more continuous edges than other methods.

Figure 2 presents the results of another experiment using the color ''lena'' image under noiseless environment. Figure 2a shows the original color ''lena'' image. Figures 2b–2e show the detection outputs of the vector gradient, MVD, entropy, and proposed edge detectors, respectively.

From Figure 2, we find that more detailed edge information is detected using the proposed method, for example, the eyes, hair, and the bottom right corner of the background. We have used the proposed method to test many other color images; the experimental

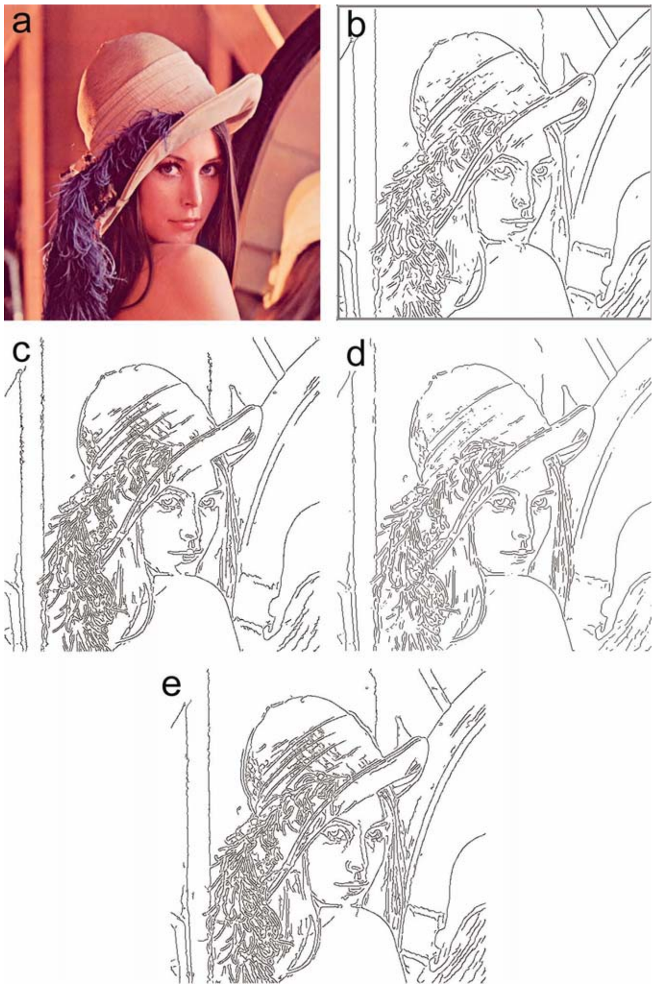

Figure 2. The original image and the edge maps of "lena" (a). The original color "lena" image (b). Gradient (c). MVD (d). Entropy (e). Proposed method. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

results also show that the proposed scheme achieved very good results with few false edges and high localization accuracies.

V. CONCLUSIONS

The color images are filtered by quaternion convolution to obtain the edges at different scales. The proposed method takes the advantage of similarities in the filter's responses at adjacent scales and then enhances the edge structures while diluting noise by multiplying the filtered results at adjacent scales using dot product. Experimental results show that the proposed method has good performance on color edge detection.

REFERENCES

P. Bao and L. Zhang, Noise reduction for magnetic resonance images via adaptive multiscale products thresholding, IEEE Trans Med Imaging 22 (2003), 1089–1099.

P. Bao, L. Zhang, and X. Wu, Canny edge detection enhancement by scale multiplication, IEEE Trans Pattern Anal Mach Intell 27 (2005), 1485–1490.

J. Canny, A computational approach to edge detection, IEEE Trans Pattern Anal Mach Intell 8 (1986), 679–698.

G. Economou, Detecting edges using density value, Electron Lett 40 (2004), 1528–1530.

L. Jin and D. Li, An efficient color-impulse detector and its application to color images, IEEE Signal Process Lett 14 (2007), 397–400.

A. Koschan and M. Abidi, Detection and classification of edges in color images, IEEE Signal Process Mag 22 (2005), 64–73.

S.C. Pei and C.M. Cheng, Color image processing by using binary quaternion-moment-preserving thresholding technique, IEEE Trans Image Process 8 (1999), 614–628.

W.K. Pratt, Digital image processing, Wiley, New York, 1991.

S. Said, N.L. Bihan, and S.J. Sangwine, Fast complexified quaternion Fourier transform, IEEE Trans Signal Process 56 (2008), 1522–1531.

S.J. Sangwine, Colour image edge detector based on quaternion convolution, Electron Lett 34 (1998), 969–971.

S.J. Sangwine and T.A. Ell, Colour image filters based on hypercomplex convolution, IEE Proc-Vis Image Signal Process 147 (2000), 89–93.

P.E. Trahanias and A.N. Venetsanopoulos, Color edge detection using vector order statistics, IEEE Trans Image Process 2 (1993), 259–264.