

3. Consider a general 2-stage and a general 3-stage explicit Runge-Kutta stencil.
- (a) Find a solution for the various Runge-Kutta coefficients such that k_1 and k_2 for both stencils coincide. Note that the b_i coefficients for each method are not necessarily the same.

- (b) Construct an adaptive stepsize solver based on these two stencils. Use it to calculate $y(1)$ for the initial value problem

$$y' = -y, \quad y(0) = 1.$$

Notice that the size of the last step has to be manually selected so the last value of x_i is exactly equal to 1. How many steps are required to achieve a error of 10^{-6} in the numerical value of $y(1)$?