

Theory : We consider an autocallable structured product with tree rebates. T1=1, T2=2, T3=3; We have that :

$$S_{T1} = S_0 \times e^{(W_1 \times \sigma + (r - \frac{\sigma^2}{2}))}$$

$$S_{T2} = S_0 \times e^{(W_2 \times \sigma \times \sqrt{2} + (r - \frac{\sigma^2}{2}) \times 2)}$$

$$S_{T3} = S_0 \times e^{(W_3 \times \sigma \times \sqrt{3} + (r - \frac{\sigma^2}{2}) \times 3)}$$

and we want to calculate that probability: $P(S_{T1} < K_1, S_{T2} < K_2, S_{T3} > K_3)$
 With K_1 : condition 1, K_2 : condition 2, K_3 condition 3.

Having that : $S_{T1} < K_1$ implies that: $W_1 < (\frac{\ln(K_1/S_0) - (r - \frac{\sigma^2}{2})}{\sigma})$

we rename $(\frac{\ln(K_1/S_0) - (r - \frac{\sigma^2}{2})}{\sigma}) = A$

Having that : $S_{T2} < K_2$ implies that: $W_2 < (\frac{\ln(K_2/S_0) - 2 \times (r - \frac{\sigma^2}{2})}{\sqrt{2} \times \sigma})$

we rename $(\frac{\ln(K_2/S_0) - 2 \times (r - \frac{\sigma^2}{2})}{\sqrt{2} \times \sigma}) = B$

Having that : $S_{T3} > K_3$ implies that: $W_3 > (\frac{\ln(K_3/S_0) - 3 \times (r - \frac{\sigma^2}{2})}{\sqrt{3} \times \sigma})$

we rename $(\frac{\ln(K_3/S_0) - 3 \times (r - \frac{\sigma^2}{2})}{\sqrt{3} \times \sigma}) = C$

$W_1, W_2, W_3 \simeq Z_1, Z_2, Z_3 \sim N(0, 1)$ Thus, we have to calculate this probability :

$$P(Z_1 < A, Z_2 < B, Z_3 > C)$$

Correlation calculus :

$$E(W_1 W_2) = 1$$

$$E(W_2 W_3) = 2$$

$$corr(W_1, W_2) = \frac{1}{\sqrt{2}}$$

$$corr(W_1, W_3) = \frac{1}{\sqrt{3}}$$

$$corr(W_2, W_3) = \frac{\sqrt{2}}{\sqrt{3}}$$

Correlation Matrix : $\Sigma =$

1	$1/\sqrt{2}$	$1/\sqrt{3}$
$1/\sqrt{2}$	1	$\sqrt{2}/\sqrt{3}$
$1/\sqrt{3}$	$\sqrt{2}/\sqrt{3}$	1

We obtain :

$$P(Z_1 < A, Z_2 < B, Z_3 > C) = \int_{-\infty}^A \int_{-\infty}^B \int_C^{+\infty} \frac{1}{\sqrt{|\Sigma|(2\pi)^3}} \times e^{(-\frac{1}{2}(x-\mu)\Sigma^{-1}(x-\mu)')} dx$$